Stuyvesant High School
Abraham Baumel, Principal

Mathematics Department
Dr. R. Rothenberg, APS

Final Examination - ME81
January 1991

Part I: Multiple choice. Answer all questions. Each question is worth four (4) points .

1. A value of k for which x-2 is a factor of $x^3 - kx^2 + x - 6$ is:
   a) 2 b) 1 c) -1 d) 4 e) -4

2. The third term of $(x^2 - \frac{1}{x})^8$ is:
   a) $8x^{14}$ b) $-8x^{12}$ c) $28x^{10}$ d) $-28x^{10}$ e) none of these

3. When written in rectangular form, $(\sqrt{2}\ \mathrm{Cis}\ 120)^4$ equals:
   a) $1+i\sqrt{3}$ b) $-1+i\sqrt{3}$ c) $1-i\sqrt{3}$ d) $-1-i\sqrt{3}$ e) none of these

4. The remainder when $5x^3 + 4x^6 - 2$ is divided by x - 1 is:
   a) 9 b) 7 c) 11 d) 5 e) none of these

5. The smallest integral upper bound of $2x^3 - 5x^2 - 7x - 10 = 0$ is:
   a) 1 b) 2 c) 3 d) 4 e) none of these

6. The graph of $r = 4\cos\theta$ is a:
   a) parabola b) line c) ellipse d) hyperbola e) none of these

7. The domain of $f = \{(x,y): y = \frac{x}{10-5x}\}$ is:
   a) $\{x: x \neq 0\}$ b) $\{x: x \neq -2\}$ c) $\{x: x \neq 2\}$ d) $\{x: x \text{ is real}\}$ e) $\{y: y \neq 0\}$

8. The inverse of $f(x) = \frac{4}{3} - \frac{2}{3}x$ is: a) $f^{-1}(x) = 2 - \frac{3}{2}x$
   b) $f^{-1}(x) = 2 + \frac{3}{2}x$ c) $f^{-1}(x) = \frac{2}{3}x + 4$ d) $f(x) = -\frac{2}{3}x - 4$ e) none of these

9. The sum of the first fifty positive even integers is:
   a) 2550 b) 5050 c) 2500 d) 1275 e) none of these

10. A polynomial equation with real coefficients that has 3 and 1+i as roots is :
    a) $x^3 + 5x^2 - 8x + 6 = 0$ b) $x^3 - 5x^2 - 4x = 0$ c) $x^2 - (4-i)x + (4-i) = 0$
    d) $x^2 + (4-i)x - (4-i) = 0$ e) $x^3 - 5x^2 + 8x - 6 = 0$

Part II: Answer all parts. Questions 11-15 are worth six (6) points each.

11. Find the coordinates of the center, and the radius of the circle described by the equation $x^2 + y^2 - 14x + 2y + 25 = 0$

12. Write the equation, in standard form, of the parabola whose vertex is at (-1,2) and focus at (3,2).

13. Find the domain and range of $y = \frac{2}{\sqrt{9 - x^2}}$

14. Find the sum and product of the roots of this polynomial

$$x^6 = 5x^2 - 10$$

15. Use DeMoivre's Theorem to find $(-\sqrt{2} + i\sqrt{2})^5$ and express your answer in rectangular form.

Part III: Answer two (2) questions from this part. Show all work! Each question is worth fifteen (15) points.

1. Prove by Mathematical Induction (15)
$$1 + 4 + 7 + \ldots + (3n-2) = \frac{n(3n-1)}{2}$$

2. Solve for all the roots of $z^3 + 8i = 0$. Place answers in a+bi form. (15)

3. Given: $2x^4 - 3x^3 - 12x^2 + 7x + 6 = 0$

A) 1. Find the maximum number of real positive roots (2)
2. Find the maximum number of real negative roots (2)

B) Find and list all possible rational roots. (3)

C) Find the integral least upper bound and integral greatest lower bound. (2)

D) Solve the equation completely over the complex numbers. (6)

4. Given the equation $25x^2 - 9y^2 - 50x = 200$

A) Find the coordinates of the vertices. (2)

B) Find the coordinates of the foci. (2)

C) Find the equation of the asymptotes. (2)

D) Find the length of the transverse axis. (2)

E) Find the length of the conjugate axis. (2)

F) Graph the curve. Label all parts. (5)

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Part I: Multiple choice. Answer all questions, ex. 1 -16. Each question is worth four (4) points. Answer on Answer sheet (Scantron paper)

1. If the third term of an Arithmetic Progression is 2 and the seventh term is (-10), then its first term is:
a) 8 b) 5 c) -1 d) -2 e) -3

2. The sum of the first 40 odd integers is:
a) 820 b) 1600 c) 1640 d) 3200 e) 3280

3. The domain of the function $y = \sqrt{x^2 - x - 6}$ is:
a) $\{x: x = -2, 3\}$ b) $\{x: -2 \leq x \leq 3\}$ c) $\{x: x \leq -3 \text{ or } x \geq 2\}$
d) $\{x: x \leq -2 \text{ or } x \geq 3\}$ e) $\{x: x \in R\}$

4. According to the rational root theorem, which of the following numbers could NOT possibly be a root of the equation
$2x^4 - x^3 + 5x^2 - 3 = 0$?
a) -1 b) $\frac{1}{2}$ c) $-\frac{3}{2}$ d) $\frac{5}{2}$ e) -3

5. How many imaginary roots does the equation $3x^8 + 5x^6 + 2x^4 - 1 = 0$ have?
a) 4 b) 5 c) 6 d) 7 e) 8

6. When $x^4 - 2x^2 + 7x - 5$ is divided by $x + 2$, then the remainder is:
a) -35 b) 9 c) 17 d) 11 e) -11

7. The sum of the roots of the equation $x^4 + x^2 - 2 = 0$ is:
a) 2 b) -2 c) 0 d) 1 e) -1

8. The coefficient of the middle term in the expansion of the binomial $(\frac{2}{x} + \frac{x}{2})^8$ is: a) 56 b) $14x^2$ c) $\frac{224}{x^2}$ d) 70 e) $448x^4$

9. The least integral upper bound of the equation
$4x^4 + x^3 - 7x^2 - 18x - 20 = 0$ is:
a) 3 b) -2 c) -3 d) 4 e) none of these

10. An equation of the parabola with vertex at the origin and focus at F(0, -1) is:
a) $y^2 = -4x$ b) $x^2 = -4y$ c) $y^2 = 4x$ d) $x^2 = 4y$ e) $x^2 = -y$

11. The quotient $\frac{(1-i\sqrt{3})^2}{(2i)^3}$ expressed in polar form is:
a) 2 cis $\frac{5\pi}{6}$ b) $\frac{1}{2}$ cis $\frac{7\pi}{6}$ c) $\frac{1}{2}$ cis $\frac{5\pi}{6}$ d) $\frac{1}{2}$ cis $(-\frac{\pi}{6})$
e) $\frac{1}{2}$ cis $(-\frac{5\pi}{6})$

12. The coordinates of the center of the hyperbola whose equation is
$x^2 - y^2 - 6x - 2y + 7 = 0$ are: a) (0,0) b) (6,2) c) (-6,-2)
d) (-3,1) e) (3,-1)

13. The equations of the asymptotes of the hyperbola whose equation is
$x^2 - y^2 - 6x - 2y + 7 = 0$ are: a) $y - 1 = \pm(x + 3)$ b) $y - 1 = \pm(x - 3)$
c) $y + 1 = \pm(x + 3)$ d) $y + 1 = \pm(x - 3)$ e) none of these

14. Which of the following is a cube root of 8?:
a) $1 - i\sqrt{3}$ b) $1 + i\sqrt{3}$ c) $-1 + i\sqrt{3}$ d) $\sqrt{3} - i$ e) $\sqrt{3} + i$

15. cis $\frac{\pi}{2}$ equals: a) 1 b) -1 c) i d) -i e) none of these

$\cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = 0 + i\cdot 1 = i$

16. $\sum_{n=0}^{\infty} \frac{1}{2^n}$ equals: a) 2 b) 1 c) $\frac{3}{2}$ d) $\frac{1}{4}$ e) is infinite

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Part II, ex. 17 - 20, 12 points each. Answer any 3 of the following 4 questions. SHOW ALL WORK

17. Given the equation $x^4 - 5x^3 + 8x^2 - 10x + 12 = 0$:
(a) Find the maximum number of positive roots 4
(b) Find the maximum number of negative roots 0
(c) List all possible rational roots $\pm(1, 2, 3, 6, 12)$
(d) Solve completely for x

18. Three numbers are in a geometric sequence with a common ratio of 3. If 3 is subtracted from the first number, 4 from the second number and 25 from the third number, the resulting numbers form an arithmetic sequence. What are the numbers in the original geometric sequence? [Only an algebraic solution will be accepted.]

19. Prove by Mathematical Induction: $5 + 10 + 15 + \ldots + 5n = \frac{5n(n+1)}{2}$

20. a) Write, in Cartesian form, the standard equation

$[(x-h)^2 + (y-k)^2 = r^2]$ of the circle whose equation given in polar form is $r = 8\cos\theta$.

b) Write an equation and sketch the graph of the parabola with focus at (5,0) and directrix $x = -5$

c) Given the ellipse $x^2 + 4y^2 = 16$, find
i) the coordinates of the major vertices
ii) the coordinates of the minor vertices
iii) the coordinates of the foci
iv) its eccentricity
v) sketch its graph

REMINDER: SHOW ALL WORK

$x, 3x, 9x$ ° $x-3, 3x-4, 9x-25$

$(3x-4)-(x-3) = (9x-25)-(3x-4)$

$3x-4-x+3 = 9x-25-3x+4$

$2x-1 = 6x-21$

$4x = 20$

$x = 5$

$5, 15, 45$; $4, 11, 20$

dif = 4